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STOCHASTIC GEOLOGICAL MODELLING USING IMPLICIT BOUNDARY SIMULATION

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ABSTRACT

Geological modelling aims at defining the spatial extent of geological units (GU) in a mineral deposit, from fragmentary information (drill hole data and surface mapping). To this end, the implicit surface or boundary modelling (ISM) is a novel approach in the mining industry imported from the computer graphic field. The ISM application to model the boundary of a GU is the following: for every drill hole sample, calculate the distance to the nearest sample belonging to another unit, assign a positive sign if the sample belongs to the target unit and a negative sign otherwise, then interpolate the signed distances in the 3D space using radial basis functions. Finally the zero-distance iso-surface (representing the boundary of the target GU) is extracted and transformed into a wireframe representation.

This work presents a method combining implicit surface modelling and existing geostatistical approaches to generate an uncertainty model for a geological unit. The proposed method simulates the distance to the geological boundary in the study area, conditionally to the information at the data locations, in order to get the simulated GU by considering the zero-distance iso-surface. The variogram parameters and the distribution of distances can be derived from a reference geological model so that the simulated GU reproduces desired features. The proposed approach is applied to a data base from the Rosario Oeste deposit and compared to a traditional approach. Relevant aspects of ISM application in the geological modelling framework versus its original field of application are discussed.

INTRODUCTION

The geological modelling process consists in defining the spatial extension of geological units related to a petrophysical property such as lithology, alteration or mineral zone. To this end, one can distinguish between two main approaches:

- Deterministic modelling, which provides a unique geological model to describe the deposit. The most common method is hand contouring and direct digitization of the geological unit boundaries.
- Stochastic modelling, which uses geostatistical methods such as sequential indicator, truncated Gaussian and plurigaussian simulation in order to generate several plausible scenarios of the geological setting. This approach allows determining spatial uncertainty measures and aspects such as selectivity-dilution, optimal grid spacing and information effect, among others.





In the scope of deterministic modelling, a new approach called implicit boundary modelling uses functions based on distance to extract the surface or boundary between different geological units. In recent years, implicit boundary modelling has been gaining popularity in the mining industry because of the easy generation and rapid update of geological models, as well as the repeatability of the process. The attractive features and the mathematical background formulation of implicit modelling make it interesting to explore and expand to the stochastic modelling approach, by combining implicit modelling and geostatistical methods.

OVERVIEW OF CURRENT GEOLOGICAL MODELLING APPROACHES

Hand Contouring-Direct modelling

It consists in defining polygons to represent the extension of the geological units, based on drill hole data projected into planes such as vertical sections and plan views. Then, one generates a 3D wireframe that honours the contacts of the units at the drill hole locations. This approach allows the explicit incorporation of geological knowledge of the deposit; however, it can be time consuming depending on the complexity of the geological units and the required resolution. Another drawback is the non-repeatability and possible geologist "bias" effect.

Implicit Modelling

The implicit boundary modelling is an imported technique from the computer graphic field, designed to generate a surface representation from a dense scattered 3D point data set, from 3D laser scanners [1]. It is important to notice that the original aim of the method is to find a surface representation of dense scattered data, whereas the goal of geological modelling is to predict the spatial extension of the geological units using a surface representation based on fragmentary and scarce data, such as drill holes and surface mapping.

The core of the method, in the case of modelling two complementary geological units, can be summarised as follows:

- For each available drill hole sample, calculate a distance to the nearest geological contact. Such a distance can be the Euclidian distance or an anisotropic distance. Assign a positive sign to the previous calculated distance if the sample belongs to the target geological unit and a negative sign otherwise.
- Interpolate the signed distance function over the domain of interest by using an exact interpolator such as radial basis functions, inverse distance interpolation or kriging.
- Extract the zero-distance iso-surface as the boundary of the target geological unit, or flag the estimated nodes depending on the sign of the interpolated distance function.

The process is illustrated on a synthetic example shown in Figure 1, using four vertical drill holes and two units.

Stochastic models

The most common methods to generate a stochastic geological model are sequential indicator simulation (SIS) [2] and truncated Gaussian simulation (TGS) [3]. Both methods rely on a codification of the geological units into indicators.







Figure 1 Example of application of implicit boundary modelling: (a) drill hole data; (b) distances calculated in samples and interpolated distances; (c) final geological model after truncation

Sequential Indicator Simulation (SIS)

This method rests on the estimation of the conditional distributions of the geological unit indicators at each target node, by means of indicator kriging [4], using the sample data and the previously simulated nodes as conditioning information. From the conditional distributions, a unit is then drawn by Monte Carlo simulation. The main advantages of the method are the simple incorporation of hard and soft data and the possibility to express spatially continuous patterns. As a counterpart, indicator kriging suffer from mathematical inconsistencies such as order relation violations, which need to be corrected [5].

Truncated Gaussian Simulation (TGS)

This method relies on the truncation of a Gaussian random field (GRF) in order to generate realisations of geological units. The main features are the reproduction of the indicator variograms associated with the geological units and a hierarchical contact relationships between units. This method is therefore adequate for deposits where the units exhibit a hierarchical spatial distribution, such as depositional environments or sedimentary formations. Plurigaussian simulation [6] is an extension of truncated Gaussian simulation that incorporates two or more Gaussian random fields and a truncation rule and allows reproducing complex contact relationships between the geological units.

IMPLICIT BOUNDARY SIMULATION (IBS)

Implicit boundary simulation has been explored as a global uncertainty model, based on the available data [7] or on a reference model to infer parameters [8]. The proposed simulation approach is presented for both options.

Implicit Boundary Simulation From Available Data

If the available data is dense enough to infer the spatial configuration of the geological units, the following approach can be used:

• Calculate the distance of each sample to the nearest contact



- Transform the calculated distances into normal scores
- Perform variogram analysis of the transformed distances
- Simulate the transformed distances, conditionally to the available samples, by using a multivariate Gaussian simulation algorithm [9, 10]
- Truncate the resulting realisations using the Gaussian value associated with the zero distance.

As an illustration, Figure 2 presents an example showing two realisations of the transformed distances (top) and the associated categorical realisations after truncation (bottom).



Figure 2 Example of implicit boundary simulation, showing simulated Gaussian distance fields (top) and categorical realisations (bottom)

Implicit Boundary Simulation Using a Reference Model

When the available data are scarce or one wants to impose the properties of a certain phenomenon to the realisations, a reference model (RM) can be used. The RM can be derived from a picture or an existing geological model. In this case, the following approach is used:

- In the reference model, calculate the distance D^{RM} of each node to the nearest contact
- Transform the calculated distances into normal scores. Store the RM transformation function and the Gaussian value associated with the zero distance.
- Perform variogram analysis of the transformed distances
- In the sample data base, calculate the distance D^{sample} of each sample to the nearest contact. Since the data base represents only a part of the reality, D^{sample} is actually the maximum possible distance to the nearest contact, and the true distance to the contact (D^{true}) can range from 0 to the distance calculated from the samples:

$$D^{true} \in [0, D^{sample}]$$



- Using the transformation function and variogram determined with the reference model, simulate the true distances D^{true} of the samples to the nearest contact, conditionally to the previous interval constraint. This can be achieved using an iterative algorithm known as the Gibbs sampler [6, 11].
- Simulate *D*^{true} over the domain, using a multivariate Gaussian simulation algorithm
- Truncate the resulting realisations to obtain the simulated geological units.

By repeating this procedure, a set of realisations reproducing the indicator variogram and proportion of the reference model, as well as the known geological units at the sample locations, can be generated.

APPLICATION TO A MINING DATA SET

Presentation of the Data

Implicit boundary simulation is applied to the mineral zones of the Rosario Oeste deposit, owned by Compañia Minera Doña Ines de Collahuasi (CMDIC), which is a structurally controlled high sulphidation system. A set of 53,735 diamond drill hole samples, composited at a length of 2m and located in a volume of $175 \text{ m} \times 2000 \text{m} \times 700\text{m}$, are available, with information of the mineral zones and total copper grades (Figure 3a). The main mineral zones in the area of study are:

- Pyritic primary: a barren unit with low copper content
- Sulphide zone: a mineralised unit composed by primary and secondary sulphides ore. Its geometry is highly controlled by geological structures (veins, faults and massive sulphide veins) with mineralisation of chalcocite, minor bornite and chacopyrite and/or enargite with pyrite, associated to a late event of the same hydrothermal system responsible of the copper porphyry mineralisation of Rosario.

The connectivity of sulfhide unit has been confirmed by trenches. This connectivity and sinuosity is reflected in the plan interpretation shown in Figure 3b.



Figure 3 a, perspective view of drill hole samples; b, plan view of the interpreted sulphide zone

Implicit Boundary Simulation



The implicit simulation approach uses the available samples to calculate the distribution of distances to the nearest contacts and the variogram of their normal score transforms. This approach is adopted because the amount of information is enough to infer those properties. Figure 4 presents the normal score variogram, which shows a smooth behaviour near the origin. This is explained by the spatially continuous nature of the distance-to-contact variable. The variogram model considers nested Gaussian structures and a very small (0.1% of the sill) nugget effect to avoid numerical instabilities.



Figure 4 Variogram of transformed distances along the main directions of anisotropy (N80°E, N10°W and vertical)

It is important to point out that the calculated distances present a trend to increase towards the outer part of the deposit (non-stationary feature); however this effect is reduced by the Gaussian transformation. Figure 5 presents two realisations superimposed with the conditioning drill hole data. It is possible to appreciate the effect of the smooth variogram model, which implies the existence of regular boundaries for the simulated sulphide unit.. Also the resulting realizations resemble the geological interpretations generated by the CMDIC geology team



Figure 5 Realisations of mineral zones at Rosario Oeste using implicit boundary simulation





PERFORMANCE EVALUATION

To evaluate the performance of implicit boundary simulation, a comparative analysis against sequential indicator simulation is presented. It consists of a cross-validation exercise, in which one removes one drill hole at a time and then simulates its samples conditionally to the remaining drill hole data. For both methods, the same simulation parameters (search distances and amount of conditioning data) are used and 25 realisations are generated. This way, it is possible to compare each realisation with the true geological unit at every drill hole sample.

For implicit boundary simulation the distances to the nearest contact and Gaussian transformation function are recalculated when a drill hole is extracted, so as to avoid a "memory" effect of the removed drill hole. The indicator variogram associated with the sequential approach is presented in Figure 6; the fitting uses a nugget effect and nested spherical models.



Figure 6 Variogram of sulphide indicator along the main directions of anisotropy (N80°E, N10°W and vertical)

Proportion of Sulphide Unit

Figure 7 shows the cumulative distribution functions of the sulphide proportion, for each method (SIS and IBS) and for the reference data. IBS gives proportions that are slightly closer to the true one.







Figure 7 Distribution of sulphide proportion for each method and for the samples

Match Percentage of Geological Unit

For each realisation, one can calculate the percentage of match between the true geological unit and the simulated unit at each sample. Figure 8 presents the match percentage distribution over the realisations for both methods. IBS exhibits a consistently higher performance (~66% of match) than SIS (~62% of match), implying a better prediction of the geological units.



Figure 8 Match percentages between true and simulated geological units

Variogram Reproduction

Figure 9 displays the expected down-the-hole indicator variogram (average indicator variogram over the realisations) for both methods and for the data values. IBS better reproduces the shape of the true indicator variogram, while SIS yields to a greater nugget effect.



Figure 9 Expected down-the-hole sulphide indicator variograms, for both simulation methods and for the samples

Sulphide Interval Length Distribution



For each realisation, one can measure the lengths of the drill hole intervals corresponding to the sulphide unit. The distribution of such lengths is then compared against the true length distribution calculated from the sample data. It is seen (Figure 10) that IBS yields a length distribution closer to the true distribution. For example 98% of the total meters of sulphide belong to intervals greater than 10m, whereas 95% of the simulated intervals by IBS are greater than 10m and just an 85% in the case of SIS.



Figure 10 Distribution of contiguous meters of sulphide unit for IBS, SIS and samples

Discussion

The overall performance of IBS is better than that of SIS, especially in relation to the sulphide indicator variogram and to the interval length distribution. The regular boundaries and connected patterns resulting from IBS stem from the spatial regularity of the distance variable, for which a Gaussian variogram model has been used.

In addition to the geological unit, the implicit approach provides the distance to the nearest contact, which conveys information about the configuration of the geological unit. This fact could improve the prediction.

CONCLUSIONS

A simulation method for categorical variables has been presented, which combines geostatistical methods and an implicit boundary modelling approach. It could be adequate for geological scenarios where the spatial regularity of boundaries and connectivity of the units are important factors and cannot be directly achieved by traditional geostatistical methods.

The implicit boundary approach provides great flexibility in terms of the distance function used and in terms of incorporation of soft data. It can be extended to more than two geological units, following a hierarchical process, which deserves further studies.

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