CONDITIONAL CO-SIMULATION OF COPPER GRADES AND LITHOFACIES IN THE RÍO BLANCO – LOS BRONCES COPPER DEPOSIT

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ABSTRACT

Geostatistical simulation is widely used to generate realisations of the spatial distribution of mineral grades in ore deposits. At present, the most common approach is to divide the deposit into rock-type domains (lithofacies) and to simulate the grades within each domain separately. The rock-type model can be obtained by a geological interpretation of the deposit and be deterministic, or be simulated prior to grade simulation. Even though this 'cascade' approach allows establishing an uncertainty model for the mineral resources in the deposit, it implicitly assumes a lack of stochastic dependence between the grades across rock-type boundaries and does not fully account for the spatial relationship between the grades and the occurrence of given rock types.

This work presents a method that allows simultaneously simulating mineral grades and rock types and taking into account their spatial dependence by using a combination of the multi-Gaussian model (for simulating grades) and truncated Gaussian model (for simulating rock types). The method is able to incorporate hard data (assays and geological logging from drill hole or blast hole samples) as well as prior geological knowledge as conditioning information for the realisations of both grades and rock types. It is applied to the Río Blanco – Los Bronces porphyry copper deposit to co-simulate copper grades and the occurrence of tourmaline breccia, and it is compared to traditional approaches against production data.

INTRODUCTION

The uncertainty associated with the recoverable tonnages and grades in a mineral deposit is a key factor in the decision-making process of a mining project. Currently, the most common approach to model the uncertainty in the spatial distribution of mineral grades is the following [1, 2]:

- Define the spatial extension of the rock type domains (lithofacies). This is usually done by either of the two methods:
  - Deterministic modelling, consisting of an interpretation of the relevant lithofacies using the available information and geological knowledge of the deposit.

- Stochastic modelling, consisting of simulating the occurrence of each lithofacies. Several methods can be used to this end, including truncated Gaussian simulation [3, 4], plurigaussian simulation [5, 6] or sequential indicator simulation [7, 8]. The outputs are several realisations (alternative models) of the spatial distribution of lithofacies.

- Simulate the mineral grades within each lithofacies conditionally to the data belonging to this lithofacies only.

From the above steps, the only relationships between the grades and the occurrence of a lithofacies are the membership of the grade data to the lithofacies and the spatial domain where the simulation of grades takes place. Accordingly, the following aspects can be identified:

- The previous approach assumes a stochastic independence between lithofacies occurrences and grade values, which is often a simplification of reality. This can be illustrated by the following trivial but evocative example. Given a sample of a highly mineralised rock type, it is probable that the grade of the sample is high. Reciprocally, a sample with a high grade value but no geological logging is likely to belong to the highly mineralised rock type

- The deterministic lithofacies modelling approach provides just one interpretation of the geology of the deposit without offering any measure of the uncertainty in the lithofacies boundaries

- The incorporation of external or soft data with information on the lithofacies occurrence has no effect on the grade realisations, except for a possible modification of the spatial extension of the lithofacies

- Since grades from different lithofacies are assumed independent, the grade realisations exhibit discontinuities at the boundaries between lithofacies, a feature that is not necessarily true in the available data.

To improve the decision-making process in the mining industry, it is of interest to account for the spatial relationships between lithofacies and grades and to incorporate geological knowledge that would exert an active role in the simulated grade values. To this end, this paper presents a methodology that links two known geostatistical models: the multi-Gaussian model for simulating mineral grades and the truncated Gaussian model for simulating lithofacies. Several proposals [9, 10, 11, 12] have already been made in this direction. However, most of them consider the simulation of grades conditioned to the lithofacies, but not the lithofacies conditioned to grades, or ignore the cross-correlation between grades and lithofacies occurrences.

METHODOLOGY

Simulation of Grades

The proposed method uses the well-known multi-Gaussian framework to simulate the mineral grades. The workflow is as follows [1]:

- Normal score transformation of the grade data into standard Gaussian values
- Variogram analysis of the transformated data
- Multi-Gaussian simulation over the domain using the transformed values as conditioning

data [13]
- Back-transformation of the simulated Gaussian values into grade values.

Simulation of Lithofacies

The second algorithm used in the proposed method is truncated Gaussian simulation (TGS), in which the lithofacies (codified as a categorical variable) is obtained by thresholding an underlying Gaussian random field. The workflow is [5]:

- Determine the lithofacies proportions and contact relationships. Summarise this information in a truncation rule (thresholds to apply to the underlying Gaussian random field)
- Model the covariance function of the Gaussian random field via the fitting of the lithofacies indicator variograms
- Generate a set of Gaussian values at the data locations that are consistent with the lithofacies coding and the modelled covariance function. This step is performed with the Gibbs sampler algorithm [14]. Because the relationship between lithofacies indicators and Gaussian values is not bijective, several realisations should be considered for the next steps
- Perform multi-Gaussian simulation using the Gaussian values of the previous step as conditioning data
- Truncate the realisations according to the truncation rule.

Joint Simulation of Grades and Lithofacies

Keeping in mind the multi-Gaussian and truncated Gaussian simulation workflows, the proposed method is supported by the following aspects:

- Multi-Gaussian simulation uses a Gaussian random field (Y_{grd}) to construct realisations of grades
- Truncated Gaussian simulation (TGS) also uses an auxiliary Gaussian random field (Y_{lith}) to construct realisations of lithofacies
- The procedure to co-simulate two or more Gaussian random fields is well established in the multi-Gaussian framework [1]
- The normal score transformation and truncation rule can be understood as two quite similar transformation procedures, from a non-Gaussian variable to a Gaussian one. Therefore TGS converts a discrete problem (simulation of lithofacies) into a continuous one by using the truncation rule and the Gibbs sampler algorithm.

The key idea of the proposed approach is to link the previous two methods by cross-correlating the Gaussian random fields Y_{grd} and Y_{lith}. The workflow to jointly simulate grades and lithofacies is:

- Determine the lithofacies proportions and contact relationships. Summarise this information in a truncation rule
- Transform the grade data into Gaussian values (Y_{grd})
- Fit a coregionalisation model for Y_{grd} and Y_{lith}. The covariance model of Y_{grd} is obtained from the transformed grade data. The covariance model of Y_{lith} is obtained by fitting the lithofacies indicator variograms. The cross-covariance between Y_{grd} and Y_{lith} is obtained by fitting the cross-variograms between transformed grade data and lithofacies indicators

- Generate a set of simulated Gaussian values for Y_{lith} at the data locations that are consistent with the lithofacies coding and the coregionalisation model. This step is performed using the Gibbs sampler, modified in order to consider Y_{grd} as a covariate
- Perform multi-Gaussian co-simulation of Y_{grd} and Y_{lith} using the Gaussian values obtained at the previous steps as conditioning data
- Back-transform Y_{grd} to obtain the simulated grades
- Apply the truncation rule on Y_{lith} to obtain the simulated lithofacies.

This way, non-independent realisations of grades and lithofacies are simultaneously generated by taking advantage of the coregionalisation model.

Incorporation of Extra Geological Knowledge

It is common to know (with a certain confidence level) about the occurrence of a lithofacies even if no sample is available. This geological knowledge can be incorporated into the proposed method using *control points* that register the lithofacies expected by experts in the area. These control points have no information about grades, leading to a *heterotopic* dataset (with more data on lithofacies than on grades). The workflow remains unchanged, insofar as the Gibbs sampler and co-simulation can be adapted to heterotopic cases. The additional information about lithofacies will exert control over the grade realisations.

APPLICATION

Presentation of the Data

The area under study is part of the Río Blanco – Los Bronces porphyry copper deposit, a breccia complex located in the Chilean central Andes. A set of 2,376 diamond drill hole samples, located in a volume of 400 m × 600 m × 130 m, are available with information on rock types and total copper grades. The main lithofacies are [15]:

- *Granodiorite (GDT)* located in the eastern and southern parts of the area. It is one of the host rocks of the breccia complex
- *Tourmaline breccias (BXT)* located in the central part of the area. It consists of GDT clasts surrounded by matrix cement dominated by tourmaline and sulphides (chalcopyrite, pyrite, molybdenite and minor bornite). The rock emplacement is related to the main alteration-mineralisation event of the breccia complex
- *Other breccias (OBXT)* outcropping in the western and southern parts of the sampled area. This group comprises different types of breccias with textural and compositional variations.

The lithofacies controls the copper grade distribution. BXT is the highly mineralised lithofacies, whereas GDT and OBXT have low copper contents. Maps of the copper grade and lithofacies data are presented in Figure 1. Near the boundaries between lithofacies, copper grades show gradational transitions, a feature that is usually found in disseminated deposits or in diffusive processes.

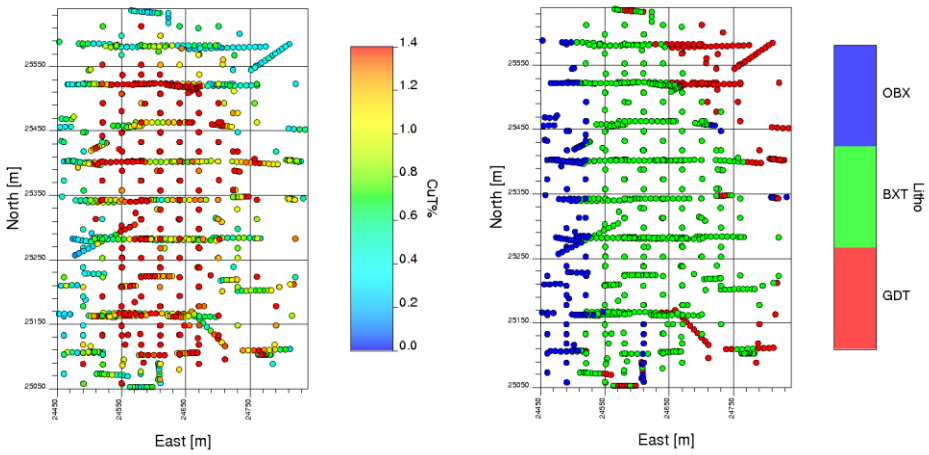

Figure 1: Location map of copper grade (left) and lithofacies (right) data

In order to simplify the start-up and inference, the lithofacies are grouped into just two types: BXT and NoBXT (GDT + OBXT).

Simulation Approaches

Six different methods will be compared.

- Copper simulation without geological control **(M1)**: this method uses the available copper grade data without considering the lithofacies information
- Joint simulation of copper grades and lithofacies **(M2)** using the proposed approach
- Joint simulation of copper grades and lithofacies incorporating additional geological knowledge on the occurrence of BXT and NoBXT. Two subcases are defined: incorporating 5% (**M3**) and 30% (**M4**) of additional data at control points that mainly belong to the NoBXT unit (in the borders of the area under study)
- Simulation of copper grades using a stochastic geological model **(M5)**: the occurrences of BXT and NoBXT are obtained using the TGS approach, and then copper grades are independently simulated within each simulated domain
- Simulation of copper grades using a deterministic geological model **(M6)**: this method uses an interpreted model of the BXT extension. Copper grades are simulated independently within the BXT and NoBXT domains

The methods, features and colours used in the next figures are presented in Table 1.

Table 1: Method names, features, colours and code names

Code	Colour	Short Name	Lithofacies	Grades	Additional Knowledge
M1	Red	No geological control	-	Simulation	-
M2	Blue	Joint simulation	Co-simulation		-
M3	Green	Joint simulation	Co-simulation		5%
M4	Pink	Joint simulation	Co-simulation		30%
M5	Black	Independent simulation	Simulation	Simulation	-
M6	Yellow	Independent simulation	Deterministic (interpreted)	Simulation	-

Implementation

For each approach, 100 realisations are generated on a grid of 390 × 590 nodes with a 1 m × 1 m spacing, in a representative bench, using the turning bands algorithm with 1000 lines [1, 13]. The anisotropy directions for variogram analyses are the horizontal and vertical directions for every model. Figure 2 indicates the truncation rule and threshold for TGS, while Table 2 presents the parameters of the coregionalisation model between the Gaussian random fields Y_{grd} and Y_{lith}.

Table 2: Coregionalisation model for the two Gaussian random fields under consideration

Structure	**Range (m)**		**Sill Contribution**		
Type	**Horizontal**	**Vertical**	$Y_{lith} \times Y_{lith}$	$Y_{grd} \times Y_{grd}$	$Y_{grd} \times Y_{lith}$
Nugget	0	0	0	0.12	0
Cubic	60	120	0.1	0.05	-0.03
Cubic	280	4,500	0.8	0.148	-0.333
Cubic	138	350	0.1	0.507	-0.218
Spherical	22	40	0	0.238	0

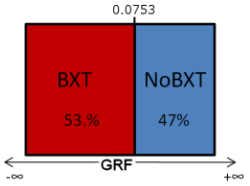

Figure 2: Truncation rule

Results

The simulated copper grades are compared on the basis of several criteria, which are presented next.

Basic statistics: Figure 3 presents the distribution of the average copper grade per realisation for each method. The distributions of the joint simulation models (M2, M3, M4) present lower values than the independent simulation models (M5, M6) and the simulation without geological control (M1). This feature is more patent as samples are added in the low-graded NoBXT lithofacies (M4), showing the influence of the additional geological knowledge in the copper grade realisations.

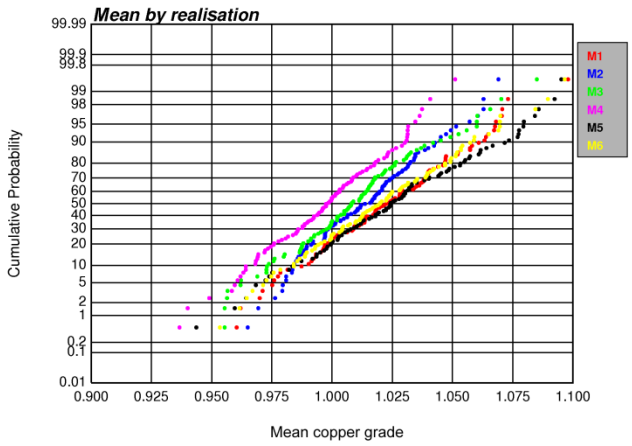

Figure 3: Distribution of average copper grade per realisation

Expected values: Table 3 presents the tonnages and mean grades above two selected cut-offs for the conditional expectation of copper grades (average of the 100 copper grade realisations). Visual and statistical inspections indicate that there is no substantial difference between the models in terms of expected values.

Table 3: Tonnages and mean grades above selected cut-offs for each model

Cut-off	**0.5% Cu**		**1% Cu**	
Model	**% of total**	**mean grade % Cu**	**% of total**	**mean grade % Cu**
M1	83.67	1.14	40.2	1.55
M2	82.52	1.14	40.34	1.53
M3	82.54	1.13	39.47	1.53
M4	82.13	1.13	39.19	1.52
M5	87.71	1.11	43.07	1.45
M6	85.95	1.12	42.46	1.47

Local uncertainty measures: Table 4 gives the basic statistics on the conditional variance of copper grades (variance of the 100 copper grade realisations). Figure 4 shows the corresponding local coefficients of variation for the joint simulation (M4) and independent simulation (M6) models. It is seen that the joint simulation model presents lower values of the uncertainty measure than the other model.

Table 4: Basic statistic**s o**n the conditional variance for each model

Model	**Min**	**Max**	**Mean**	**Std. Dev**
M1	0	4.231	0.236	0.433
M2	0	4.005	0.208	0.375
M3	0	4.050	0.206	0.372
M4	0	4.008	0.198	0.368
M5	0	4.509	0.281	0.377
M6	0	4.501	0.264	0.380

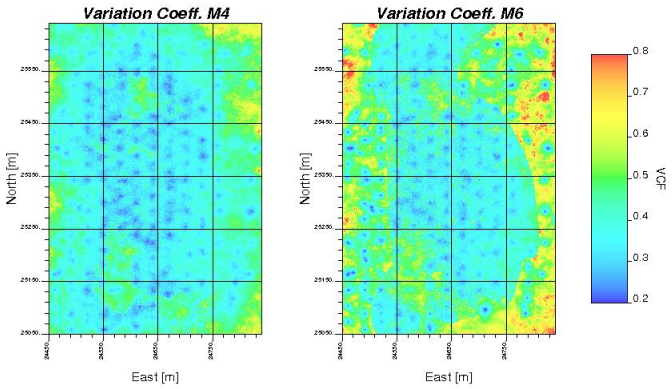

Figure 4: Local coefficients of variation for joint simulation (M4, left) and independent simulation (M6, right) models

Performance Evaluation

Given a set of 20,893 blast hole data in which the copper grades are known, a validation model is created by ordinary kriging on 5 m × 5 m blocks in the bench where the copper grade realisations were generated. Every realisation is then regularised to that block size and compared against the validation model.

Error distribution: The percentual mean error (PME) is defined by:

$$PME(k) = \frac{1}{n}\sum_{i=1}^{n} 100 * \frac{[S^k(i) - R(i)]}{R(i)} \quad (1)$$

where $S^k(i)$ is the simulated grade at block i, $R(i)$ is the grade of the validation model and n is the number of blocks where both models are defined. Figure 5 (left) presents the PME distribution for each model. The joint simulation models (M2, M3, M4) have a PME distribution closer to zero than the independent simulation models (M5, M6). Model M4 (with 30% of additional geological data) presents the best results.

Grade correlation distribution: The linear correlation coefficient between every realisation and the validation model is calculated. As seen in Figure 5 (right), the joint simulation models have higher correlations than the independent simulation models. Again, the best performance is achieved by the joint simulation model that includes the largest amount of additional geological information.

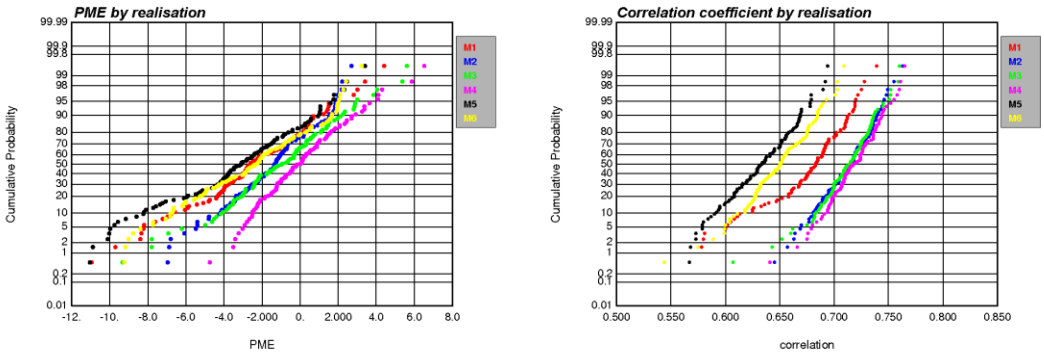

Figure 5: PME distribution (left) and correlation coefficient by realisation (right)

Destination mismatch: The percentage of correctly classified blocks (mill/dump) for several cut-off grades is calculated for each realisation. Figure 6 (left) presents the expected percentages for each model and cut-off. The joint simulation models perform consistently better than the other approaches.

Accuracy plot: Given an interval of probability p in the distribution of simulated block grades, the effective percentage of blocks of the validation model that fall within the interval is calculated. The closest this percentage to p, the better the model is in terms of accuracy and precision [16]. Figure 6 (right) displays the accuracy plots, showing a moderately better performance of the joint simulation models.

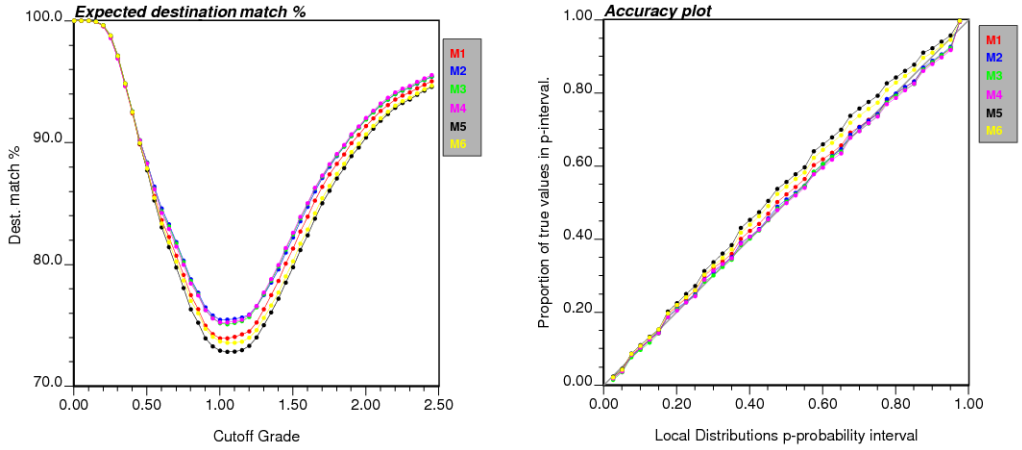

Figure 6: Percentage of correct classification (left); accuracy plot (right)

CONCLUSION

A geostatistical approach to co-simulating mineral grades and lithofacies has been presented. It is suitable when grades have gradational transitions across the lithofacies boundaries (soft geological boundaries) and allows one to incorporate additional geological information on lithofacies that is likely to influence the grade realisations.

Even if the proposed approach looks similar to other approaches in terms of expected grades, differences have been observed when looking at uncertainty measures and when validating the realisations against production data. According to these differences, is it seen that the traditional approaches overstate the uncertainty associated with the mineral resources. Stochastic mine planning approaches should therefore consider the intrinsic relationship between lithofacies and mineral grades in order to allow better decision-making for mine executives.

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